

Classes 18-20 - Meanings Skepticism and The Private Language Argument
Wittgenstein, "Meaning as Use"
Wittgenstein, "On Private Language"
Wittgenstein, from Remarks on the Foundations of Mathematics
Kripke, "On Rules and Private Languages"

II. Locke, Augustine, and Meaning as Use

Locke argued that our words must refer to our sensations in order for us to use them to communicate. Otherwise, we would not know what we are communicating.
Locke's picture of language takes communication to be like playing catch.

- B1. I hold the ball in my hands.
- B2. Then, I toss the ball.
- B3. Lastly, you catch and hold the ball

Analogously, when we communicate:

- C1. I have a sensation, which I label with a word: apple, ball, cat.
- C2. Then, I speak: The cat ate an apple.
- C3. Lastly, you associate my words with some inner sensations of your own.

The Lockean claim that our terms must refer to our ideas in order for communication to succeed appears again as Frege's claim that sense determines reference, that our inner grasp of the sense of an expression suffices to pick out its referent.

Wittgenstein's private language argument (PLA) is a direct response to the Lockean picture of communication.

There are profound disagreements about the nature, extent, and source of the PLA.

It is generally agreed that it is central to Wittgenstein's later work, the *Philosophical Investigations*.

Traditional interpretations locate the argument between §243 and §315 or so, with some further remarks later on.

Saul Kripke, in his 1982 *Wittgenstein on Rules and Private Language*, locates the argument significantly earlier in the text.

On the traditional interpretation, the argument centrally concerns the nature of mental states.

On Kripke's interpretation, the argument centrally concerns the nature of rules and rule-following.

Kripke sees the discussion of mental states as an example of a more general claim.

To distinguish between the traditional interpretation of the PLA and Kripke's version, philosophers have taken to calling the author of the latter Kripkenstein.

Wittgenstein does not attribute the Lockean picture to Locke, but to Augustine, in the selection "[Meaning as Use](#)" which begins the *Investigations*.

In §1, Wittgenstein considers a slip of paper with the words 'five red apples', used in a grocery.

We observe that while 'apple' works the way Augustine and Locke say it does, 'five' and 'red' seem different.

They do not represent objects.

They provide instructions for action.

We might, as Plato did, reify redness or the number five.
Nevertheless, the uses of those terms are different from the uses of 'apple'.

When we use a word, we follow conventional guidelines, like the grocer comparing the term 'red' with a patch of red, and counting.

Such terms require instructions for how to use them.

The same object, under different instructions, may have a different label.

Consider the way a child's stick can be a rocket, or a sword, or a pony.

Or, consider this old [Saturday Night Live skit](#).

Wittgenstein points out that the meanings of the terms 'brake' and 'lever' depend on the uses we make of the objects to which they refer.

"I set the brake up by connecting up rod and lever." - Yes, given the whole rest of the mechanism. Only in conjunction with that is it a brake-lever, and separated from its support it is not even a lever; it may be anything, or nothing (*Philosophical Investigations* §6).

Wittgenstein here anticipates Quine's semantic holism.

Terms of language have no meaning apart from their use in a larger theory.

I. Starting with Language

Let's jump to §304, where Wittgenstein is considering terms for our sensations.

"But you will surely admit that there is a difference between pain-behavior accompanied by pain and pain-behavior without any pain?" - Admit it? What greater difference could there be? - "And yet you again and again reach the conclusion that the sensation itself is a *nothing*." - Not at all. It is not a *something*, but not a *nothing* either! The conclusion was only that a nothing would serve just as well as a something about which nothing could be said (*Philosophical Investigations* §304).

Wittgenstein takes pain as a paradigmatic sensation, but you can think of your impression of a color, or an odor or taste just as well.

Locke started his analysis of language with the presumption of the existence of sensations.

Our conscious mental states are the sole things of which we are directly aware, as Descartes insisted.

From our sensations, we construct a language which refers to them.

For Locke, and Hume, and the logical empiricists, sensations are the basis on which all of our knowledge is built and the grounds for all of the rest of our language.

Even Quine argued that the boundary conditions on our theory construction are our sense experiences.

Wittgenstein works in the other direction.

Starting with language, and working backwards to its grounds, we begin to doubt the sensations which Locke and all the rest took for granted.

Thus, Wittgenstein starts the *Investigations* with an analysis of some simple languages, and some simple games, and tries to account for our more complicated, more complete, languages.

The *Investigations* begins with the claim that there are lots of different kinds of uses of language, lots of different language games.

But how many kinds of sentence are there? Say assertion, question, and command? - There are *countless kinds*: countless different kinds of use of what we call “symbols”, “words”, “sentences”. And this multiplicity is not something fixed, given once for all; but new types of language, new language-games, as we may say, come into existence, and others become obsolete and get forgotten...

Here the term “language-game” is meant to bring into prominence the fact that the *speaking* of language is part of an activity, or of a form of life (*Philosophical Investigations* §23).

The question with which Wittgenstein starts concerns the relations among these language games. Recall Ben’s point that Strawson is looking at language in its native habitat. This remark is even more aptly applied to Wittgenstein. Indeed, Strawson was deeply influenced by Wittgenstein. Wittgenstein rejects the earlier, logical view of language.

Here we come up against the great question that lies behind all these considerations. - For someone might object against me: “You take the easy way out! You talk about all sorts of language-games, but have nowhere said what the essence of a language-game, and hence of language, is: what is common to all these activities, and what makes them into language or parts of language. So you let yourself off the very part of the investigation that once gave you yourself most headache, the part about the *general form of propositions* and of language.”

And this is true. - Instead of producing something common to all that we call language, I am saying that these phenomena have no one thing in common which makes us use the same word for all, - but that they are *related* to one another in many different ways. And it is because of this relationship, or these relationships, that we call them all “language” (*Philosophical Investigations* §65).

In the *Tractatus*, Wittgenstein had, like Augustine, been looking for a single account of the nature and application of language, like Russell’s ideal, logically perfect language, or Frege’s *Begriffsschrift*, one in which all of the truths could be written.

Wittgenstein was moved, one story goes, to abandon his earlier project by trying to analyze the logical structure of an obscene gesture made by the Italian economist Piero Sraffa.

Wittgenstein’s argument is that if we can get clear about what the uses of our language are, and how we learn to use language, we can get clear about meaning and reference.

We can stop thinking of ‘five’ and ‘red’ as terms for objects, and start thinking of them according to the uses they prescribe.

IV. Language Games and Rules for Use

In §1, Wittgenstein considered a language with (at least) three terms: ‘five’, ‘red’, and ‘apple’.

He conceded that ‘apple’ might work the Augustinian way, as the name or an object.

But ‘red’ and ‘five’ are directions for actions.

Let’s return to §8-§11, where Wittgenstein extends the activities of the tribe using the slab language, to include counting, and other functions.

The different functions are all related, as all tools are related: loosely.

That terms function differently does not preclude that they stand for objects.

One can say that the signs “a”, “b”, etc. signify numbers; when for example this removes the mistaken idea that “a”, “b”, “c”, play the part actually played in language by “block”, “slab”, “pillar”. And one can also say that “c” means this number and not that one; when for example this serves to explain that the letters are to be used in the order a, b, c, d, etc. and not in the order a, b, d, c (*Philosophical Investigations* §10).

Wittgenstein is claiming that our number terms are primarily directions for how to proceed. We use them for counting apples, for example. In addition, there are directions for how to proceed using the number terms alone. If we want to understand the number terms, then we have to analyze how these terms function. In particular, we have to determine how we learn the rules for counting. As far as we agree on the rules for counting, we have some common language, we can communicate. If we were to disagree on the rules, we would not understand each other.

If a lion could talk, we could not understand him (*Philosophical Investigations*, p 223).

The meanings of terms like ‘red’ and ‘five’ consist in the rules for playing the games with those terms, for using them.

III. Wittgenstein’s Skeptical Puzzle about Counting

Let’s take some time to consider the skeptical puzzles in §143 and §185. In these cases, someone deviates from normal counting rules. We want to say that the deviant counter has made a mistake. Wittgenstein challenges us to justify our claim. If number terms worked like object terms (e.g. ‘apple’), we could perhaps explain our use, against the deviant’s use, by pointing. We have already seen, though, that number terms function differently from the Locke/Augustine account. In addition, pointing itself can not settle any matter.

Now one can ostensibly define a proper name, the name of a colour, the name of a material, a numeral, the name of a point of the compass and so on. The definition of the number two, “That is called ‘two’” - pointing to two nuts - is perfectly exact. - But how can two be defined like that? The person one gives the definition to doesn’t know what one wants to call “two”; he will suppose that “two” is the name given to *this* groups of nuts! - He *may* suppose this; but perhaps he does not. He might make the opposite mistake; when I want to assign a name to this group of nuts, he might understand it as a numeral. And he might equally well take the name of a person, of which I give an ostensive definition, as that of a colour, of a race, or even of a point of the compass. That is to say: an ostensive definition can be variously interpreted in *every* case (*Philosophical Investigations* §28).

The very act of pointing is already an action with rules for use. We could point by tracing the line from the tip of our finger backwards (§185). Here again, we can see Wittgenstein’s rejection of the *Tractatus*, which depended on an unambiguous ability to point.

Wittgenstein is trying to undermine the view that words have any sort of meaning independently of how we use them.

Moreover, he believes that uses are not determined independently of our practices.

They have no intrinsic or transcendent meaning.

No terms could have more unambiguous, universal meanings than number terms.

But even the rules governing our uses of number terms are subject to our interpretation.

We may be misled by the kinds of universal agreement that some claims engender into thinking that some words have transcendent meaning.

But if we reflect on the nature and source of that agreement, we find no grounds for the claim that words have meanings outside of our uses of them.

“But *are* the steps then *not* determined by the algebraic formula?” - The question contains a mistake (*Philosophical Investigations* §189).

In *Remarks on the Foundations of Mathematics*, Wittgenstein considers the language games of a community of wood-sellers who use number terms differently than we do.

They measure quantities of wood by surface area rather than by volume.

How could I shew them that - as I should say - you don't really buy more wood if you buy a pile covering a bigger area? - I should, for instance, take a pile which was small by their ideas and, by laying the logs around, change it into a 'big' one. This *might* convince them - but perhaps they would say: “Yes, now it's a *lot* of wood and costs more” - and that would be the end of the matter. - We should presumably say in this case: they simply do not mean the same by “a lot of wood” and “a little wood” as we do and they have a quite different system of payment from us (*Remarks on the Foundations of Mathematics I* §150)

The oddity of Wittgenstein's example is tricky to characterize.

We are tempted to complain that Wittgenstein is ignoring certain transcendent facts about mathematics.

Wittgenstein claims that we are just blinded to the conventional nature of our uses of language by the fact that our community uses term in particular ways.

Consider our attempts to establish permanent markers of nuclear waste; see p 29 of [Permanent markers](#); for the [Sandia report](#), see p 150 et seq. and p 262 et seq.)

It is difficult to establish transcendent meanings.

For Wittgenstein, there could be no hope of communicating danger to a community completely disconnected from our own.

The meanings of our signs are connected to the practices for which we use them.

Attempts to construct permanent, transcendent markers may be quixotic.

In response to Wittgenstein's skepticism about meaning, we might respond that what grounds the correctness of one way of counting, over others, is something in our thoughts.

If numbers are objects of thought (and thus work like apples) then we can apprehend, correctly or not, the next number in a series.

The series is already complete.

But, if numbers are rules for proceeding, it's not obvious how to justify going on.

How is it decided what is the right step to take at any particular stage? - “The right step is the one that accords with the order - as it was *meant*.” - So when you gave the order +2 you meant that he was to write 1002 after 1000 - and did you also mean that he should write 1868 after 1866, and

100036 after 100034, and so on - an infinite number of such propositions?...It would almost be more correct to say, not that an intuition was needed at every stage, but that a new decision was needed at every stage.

“But I already knew, at the time when I gave the order, that he ought to write 1002 after 1000.” - Certainly; and you can also say you *meant* it then; only you should not let yourself be misled by the grammar of the words “know” and “mean”. For you don’t want to say that you thought of the step...

[Y]our idea was that that act of meaning the order had in its own way already traversed all those steps: that when you meant it your mind as it were flew ahead and took all the steps before you physically arrived at this or that one (*Philosophical Investigations* §186-§188).

Wittgenstein is urging us to see that the rule is a result of a decision, rather than a pre-established, transcendent meaning.

The problem of distinguishing between the two ways of counting, ours and the deviant’s, is not merely an artifact of the way in which we represent the situation.

Both we and the deviant can be depicted as following rules, since a rule with an exception is still a rule.

This was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule (*Philosophical Investigations* §201).

We can not support our claim that the counting deviant or the woodseller is erring by claiming that s/he is not following a rule.

The claim can only be that s/he is not following our rule.

The question then becomes how we can justify imposing our rule on the deviant.

We need a rule to guide the choice of rules.

VI. Kripke’s Problem

Wittgenstein’s skeptical problems inspire Kripke’s version of Wittgenstein’s puzzle.

Kripke’s version of the paradox takes the form of two competing rules for addition, plus and quus.

Imagine that we have never actually added any terms greater than 57.

‘57’ is just a stand-in for any number greater than which we haven’t already added; there is some such number for all of us.

We all know plus.

Quus is symbolized \oplus , and is defined thus:

$$\begin{aligned}x \oplus y &= x + y, \text{ if } x, y < 57 \\ &= 5, \text{ otherwise}\end{aligned}$$

Who is to say that this is not the function I previously meant by ‘+’? (Kripke, 628)

Kripke points out that the skeptic’s claim that we meant quus, when we used plus, is crazy.

The problem is to determine why it is crazy.

There must be some fact about our past usage that makes it crazy, some fact about how we already agreed to use the language, some decision we already made.

But, in Kripke’s case, such possibilities are ruled out *ex hypothesi*.

There are pairs of numbers which I have never previously added.
For such pairs, no facts about my past use will determine how to proceed.

When asked for the answer to '68 + 57', I unhesitatingly and automatically produced '125', but it would seem that if previously I never performed this computation explicitly I might just as well have answered '5'. Nothing justifies a brute inclination to answer one way rather than another" (Kripke, 631).

Kripkenstein is considering a skeptical puzzle, the one which Wittgenstein calls a paradox in §201. There are various ways that one can respond to a skeptical puzzle. Kripke contrasts skeptical solutions with straight ones. A straight solution shows that the skeptic is wrong; in fact we do know what the skeptic denies that we know. A skeptical solution, in contrast, accepts the skeptic's claims, but shows that they aren't interesting or important. According to Kripke, Wittgenstein presents a skeptical solution to the puzzle. The solution to the skeptical problems, says Kripke, leads to the PLA.

Kripke and Wittgenstein presented two failed straight solutions: intent and pointing. In order to intend a particular method of counting, or plus rather than quus, or green rather than grue, we have to have some sort of mental representation of those intentions, some thought about which we intend. But, we have no mental representation of the rule in precisely the cases in which deviant interpretations are possible, ex hypothesi. Like intentions, appeals to pointing turned out to be question-begging, since they presupposed rules for the pointing. That is, if we could legitimately appeal to rules for pointing to determine how to follow rules for counting or measuring, we could just appeal to the rules for counting themselves. The problem is that we are wondering what justifies our uses of certain rules, including rules for pointing, over deviant ones. The interpretations of rules are precisely what are at issue, here. (Lewis Carroll has an amusing piece about how rules can not determine their own interpretations, which I've put [here](#).)

If the grounds for claiming that we mean plus, and not quus, by 'plus' are supposed to be my thinking of one rule over the other, there would be no way to defeat the skeptic. For, there are no previous thoughts to justify one over the other. Wittgenstein believes, though, that the grounds of our using plus are the ways in which we use language.

This is simply what we do (*Philosophical Investigations* §217).

The problem of my having never added a particular pair of numbers, never having thought of this particular sum, is deflated.
For, those were not the grounds for choosing plus over quus.

As a result, we have reasons to believe that the grounds for my using a term like 'five' is not an internal mental state, a private thought, the way Locke and Augustine depicted. Instead, the grounds for my using 'five' in the way that we do have to do with the way that number terms are used in public, by the community, as rules for counting.

It follows that mathematical rules are not categorical, in the way that they are traditionally conceived. There is a fundamentally conventional aspect to all of mathematics, and by extension to all of language. Private acts are, in a sense, without the force of public acts. They lack real grounds.

Why can't my right hand give my left hand money? - My right hand can put it into my left hand. My right hand can write a deed of gift and my left hand a receipt. - But the further practical consequences would not be those of a gift. When the left hand has taken the money from the right, etc., we shall ask, "Well, and what of it?" And the same could be asked if a person had given himself a private definition of a word; I mean, if he has said the word to himself and at the same time has directed his attention to a sensation (*Philosophical Investigations* §268).

One's right hand can not give a gift to one's left hand.
The privacy of the act annuls it.

V. From Rule-Following to Private Language

The plus/quus example concerns rule-following in mathematics. The PLA, as traditionally conceived, is a more general claim about language and mental states. The connection between Kripke's argument about the grounds for my mathematical claims to the grounds for my sensation claims, to the PLA as it is traditionally conceived, goes through some general claims about language and rule-following. Kripke takes Wittgenstein to argue that the meanings of my terms are the rules for my uses of those terms. Those rules are public rules; those meanings are public meanings. So, even our terms for my sensations do not refer to anything privately available to introspection. They instead refer to public criteria for meaning.

In §243 et seq. Wittgenstein makes his skeptical claim about a private language, one in which, "the individual words... are to refer to what can only be known to the person speaking; to his immediate private sensations. So another person cannot understand the language." We have moved, now, from discussing 'five' to discussing 'red' in Wittgenstein's 'five red apples'. The terms of the private language may be any terms for sensations, for things, or purported things, which are available to introspection. It seems pretty obvious that we experience sensations. We describe those sensations: a sharp pain, a bright yellow, a sweet and spicy tang. The question is whether we could have a language that consists only of private terms, or whether our ability to construct a language of these sensations is rooted necessarily in our connections to our community.

Wittgenstein considers, in §258, writing a name, 'S', for a sensation in his diary. He notes, first, that ostensive definition is inapplicable, literally, though we might metaphorically point, concentrating our attention to the sensation. Still, Wittgenstein argues, no language has been constructed.

I have no criterion of correctness. One would like to say: whatever is going to seem right to me is right. And that only means that here we can't talk about 'right' (*Philosophical Investigations*

§258).

Don't consider it a matter of course that a person is making a note of something when he makes a mark - say in a calendar. For a note has a function and this 'S' so far has none (*Philosophical Investigations* §260).

Wittgenstein's notion of a function, in §260, refers back to the problems with rules which Kripke discusses.

The sensation itself has whatever function sensations have.

We can even assume that each time we write 'S' in a calendar we are feeling some sensation.

Wittgenstein's worry is that there is no way to determine whether the same sensation is being felt each time we write 'S'.

Language demands a kind of regularity, §207.

But, there is no ground for assuming regularity among the instances in which I write 'S', since they are all private.

Every action according to the rule is an interpretation...And hence also 'obeying a rule' is a practice. To *think* one is obeying a rule is not to obey a rule. Hence it is not possible to obey a rule 'privately': otherwise thinking one was obeying a rule would be the same thing as obeying it (*Philosophical Investigations* §§201-2).

Wittgenstein's claim is that following a rule is an action, a practice, governed by public criteria, rather than a mental affirmation of a description of those actions.

Kripke thus thinks that the PLA is really a more general claim about rule following, and not the more specific claim about a language of sensations.

What is really denied is what might be called the 'private model' of rule following, that the notion of a person following a given rule is to be analyzed simply in terms of facts about the rule follower and the rule followed alone, without reference to his membership in a wider community... The impossibility of a private language...does indeed follow from the incorrectness of the private model for language and rules, since the rule following in a 'private language' could only be analyzed by a private model, but the incorrectness of the private model is more basic, since it applies to all rules (Kripke, 635).

VIII. Sensations: Not Somethings, Not Nothings

As Kripke says at the end of his essay, he neglects the application of his interpretation of the PLA to the material on sensations in §243 et seq.

One of the central questions arising from those sections concerns the distinction between Wittgenstein's methodological claims and any ontological claims that might follow from his argument.

Wittgenstein's view of the ontology of private sensations, recall, is puzzling.

It is not a *something*, but not a *nothing* either! (*Philosophical Investigations* §304).

As Wittgenstein says in §261, if one has something, there must be some thing to have.

One can not avoid the commitment by mere semantics.

Thus, if we used terms which presumed sensations, as 'apple' presumes apples, we would be committed

to taking sensations as objects.

But, our sensations do not play significant roles in our language use.

Note the confusion:

What am I to say about the word “red”? - that it means something ‘confronting us all’ and that everyone should really have another word, besides this one, to mean his *own* sensation of red? Or is it like this: the word “red” means something known to everyone; and in addition, for each person, it means something known only to him? (*Philosophical Investigations* §273)

Consider the beetle in the box, §293.

Since no one can see what is in another person’s box, the question of whether they are correct or incorrect about the behavior or characteristics of the beetle can never come up.

We can divide through, or subtract out, the beetle itself, while continuing to talk about beetles.

The language-game of beetles presupposes no beetles.

Similarly, Wittgenstein says, our language of sensations presupposes no sensations.

They lack criteria for identity.

What is the criterion for the sameness of two images? - What is the criterion for the redness of an image? For me, when it is someone else’s image: what he says and does. For myself, when it is my image: nothing. And what goes for “red” also goes for “same” (*Philosophical Investigations* §377).

At these words I form this image. How can I *justify* this?

Has anyone shown me the image of the colour blue and told me that *this* is the image of blue?

What is the meaning of the words: “*This* image”? How does one point to an image? How does one point twice to the same image? (*Philosophical Investigations* §382).

Wittgenstein’s position is odd, appropriately represented by his paradoxical-sounding claim that sensations are not somethings and not nothing.

The oddity of the claim is that the sensation itself seems to play a significant role in my communication about my sensations, whereas I can come to see the beetle as irrelevant.

To settle the claim about whether sensations are real things, we must look at how we use the terms for them.

We find that we do talk about sensations in ways that at least seem to presume their reality.

It is not merely that we have introspective awareness of our sensations.

More significantly, we discuss and compare our sensations with those of other people.

We ask questions about whether we all feel the same kind of pain.

We wonder whether there are inverted spectra (§272).

Wittgenstein (§303) says that we make decisions to raise these questions, that they do not necessarily arise.

Once we decide to talk about sensations, we seem more committed to them than we should be, §308.

One might say that they are artifacts of the grammar we adopt, if one were adopting Wittgenstein’s use of ‘grammar’.

Consider the mocking §298.

If our talk of sensations as real things is an artifact of the grammar we adopt, it is no evidence of their transcendent reality.

It is evidence of them being not a something, even if, introspectively, they are not nothing.

VII. Robinson Crusoe

Robinson Crusoe, in the books by Daniel DeFoe, was shipwrecked, living alone.

Tom Hanks' character in *Cast Away*, is a similar case.

Such persons, we think, would be able to create a language that no one else understands.

Thus, they seem to be counter-examples to Wittgenstein's claim that a private language is impossible.

In response, note that these cases are not real counter-examples.

The private language that Wittgenstein considers is not one that no one else in fact understands.

It is a language that no one in principle could understand.

The individual words of this language are to refer to what *can only be known to the person speaking*; to his immediate private sensations. So another person *cannot* understand the language (*Philosophical Investigations* §243, emphasis added).

Further, remember that Wittgenstein's claim is that the terms of our language are closely connected to their rules for use; meaning is use.

A private language would entail private rules for using that language.

Wittgenstein urges that a rule is essentially an artifact of a community.

It is not possible that there should have been only one occasion on which someone obeyed a rule. It is not possible that there should have been only one occasion on which a report was made, an order given or understood; and so on. - To obey a rule, to make a report, to give an order, to play a game of chess, are *customs* (uses, institutions) (*Philosophical Investigations* §199).

Crusoe and *Cast Away* are not counter-examples to the PLA because we understand them as essentially members of our community.

They are isolated physically, but not apart from our conventions.

X. Conventionality

Traditionally, we think of mathematics as completely independent of our interests or conventions.

Two and five are seven, independently of us.

Mathematical facts are like transcendent physical facts, like the earth's revolution around the sun and unlike the fact that we drive on the right-hand side of the road.

For Wittgenstein, controversially, mathematics is conventional.

Kripke insists that Wittgenstein does not believe that the truth of a mathematical claim depends only on popular agreement.

He cites the following, from Wittgenstein, in support of his claim:

Certainly, the propositions "Human beings believe that twice two is four" and "Twice two is four" do not mean the same (Kripke 636, *Philosophical Investigations* p 226).

This sentence seems to imply that mathematical claims are not the result of mere decisions.

But, see the rest of the original passage:

The latter is a mathematical proposition; the other, if it makes sense at all, may perhaps mean: human beings have *arrived* at the mathematical proposition. The two propositions have entirely different *uses*. - But what would *this* mean: "Even though everybody believed that twice two was five it would still be four"? - For what would it be like for everybody to believe that?
(*Philosophical Investigations* p 226)

It does look as if Wittgenstein thinks that there is something Moore-paradoxical about the traditional claim that mathematics is independent of us.

Moore's so-called paradox is an assertion of the form:

p but I don't believe that p

For example:

It is raining, but I do not believe that it is raining.

(The 'I do not believe...' is intended literally, and not as indicating that one is surprised.)

Moore-paradoxical sentences are self-refuting.

You can not both assert a sentence and sincerely not believe that sentence, on pain of contradiction.

Compare to §288, where Wittgenstein claims that we can't know what 'pain' means and then be in doubt about whether we are in pain.

Wittgenstein clearly seems to be saying that mathematics depends on convention, on popular agreement.

Wittgenstein develops his mathematical conventionalism in greater depth in *On Certainty*.

There, he argues that the apparent transcendence of mathematical claims is something like a logical fact about those sentences.

The logic is a kind of limit on our language game.

I did not get my picture of the world by satisfying myself of its correctness; nor do I have it because I am satisfied of its correctness. No: it is the inherited background against which I distinguish between true and false. The propositions describing this world-picture might be part of a kind of mythology. And their role is like that of rules of a game; and the game can be learned purely practically, without learning any explicit rules (*On Certainty* §94-§95).

Logical propositions are like Moore's 'I know that here is a hand'.

They serve as a river bed on which ordinary empirical propositions flow.

We can use them to teach the use of certain terms.

We can say that sentences like 'There are physical objects' are senseless as a way of teaching the term physical objects.

Similarly, we can say '5+7=12' as a way of teaching the rules of arithmetic, but not to say anything about objects like 5, 7, and 12.

Such bare claims are too obvious to have any content.

The only sensible use of such sentences would be, for example, to teach children their numbers, or their addition facts.

Mathematics and what we ordinarily call logic are in the river bed.

We can not even defend our knowledge of such claims.

How could you convince some one that you knew that 5+7=12?

You would have to convince them that you knew something much wider than that one proposition.

Some propositions, like the claims that there is an external world or that no human being has ever stood on the surface of the sun, are similarly nonsensical, or limiting, or logical.

Of course, we might call it a contingent fact that no one has been on the sun.

The fact that no one has stood on the sun is not a truth of reason.

But, it also is not going to change.

For Wittgenstein, rules, like those in the riverbed and those which distinguish the bed from the river, are conventional, and indeterminate.

One way to try to resolve the indeterminacy is to appeal to inner states.

We seem to know what the rules are, which propositions are bedrock, which truths are unassailable, as a matter of feeling.

But inner states are irrelevant, if we look to use for meaning.

Experience can not give us the rules, either.

The ordinary understanding of the rules is that they are learned by induction over experience.

The basic process of induction is that we see a few examples and then come to a general rule.

Wittgenstein thinks that there is a fallacy in this ordinary understanding, that we do not so much derive the general rule from the few instances, but use the rule as a way of organizing the instances.

We don't learn propositions one at a time; we take on a system as a whole.

When we first begin to *believe* anything, what we believe is not a single proposition, it is a whole system of propositions. (Light dawns gradually over the whole.) It is not single axioms that strike me as obvious, it is a system in which consequences and premises give one another *mutual* support (*On Certainty*, §141 - §142)

Again, we can see Wittgenstein's anticipation of Quinean holism.

Still, the picture that Wittgenstein provides is one in which there are basic truths, which are almost empty in that they have little use or value: people don't fly off into space, the sun is not a hole in the vault of heaven, $2+2=4$, there is a material world.

Denying these is like denying the rules of the game, changing the subject.

Sometimes, what looks like an empirical proposition turns out not to be so.

We can change our views about which sentences are like the river bed, and which ones are like the river.

Wittgenstein has come very close, in asserting the continuity of river and riverbed, to abandoning all hope for firm and permanent claims about the world.

For, if any proposition can be taken as bedrock, and any can be part of the river, it seems conventional whether we hold logic or mathematics steady, or whether we hold religious, or moral, or just crazy views as bedrock.

We might even hold idealist claims as bedrock, or skeptical ones.

IX. Conclusion

It is an open question whether Kripke correctly interprets Wittgenstein, whether the PLA is essentially a corollary of the arguments about rule following, or if the claims about a language of sensations are essentially focused on that specific case.

Kripke's case is certainly helped by Wittgenstein's claim at the end of §304:

The paradox disappears only if we make a radical break with the idea that language always functions in one way, always serves the same purpose: to convey thoughts - which may be about houses, pains, good and evil, or anything else you please.

Whether or not Kripke has correctly interpreted Wittgenstein, we can evaluate the Kripkenstein skeptical solution.

Wittgenstein appeals, if Kripke is right, to the community's values.

It followed from Wittgenstein's solution both that there are serious questions about the status of our conscious states, and that the community which sets the standards which fix the meanings of our terms has freedom to choose alternate rules.

The freedom to use alternate rules was evinced by Wittgenstein's examples of the wood-sellers.

Wittgenstein imagines lots of different ways that a community could measure wood, and fails to justify one method over the others.

Another option would be an alternative straight solution.

Kripke and Wittgenstein reject pointing and intentions.

Ruth Millikan proposes an evolutionary explanation.

We follow the rules that we do because we are hard-wired as a result of natural selection to do so.

We are justified in using plus, rather than quus, or for counting in the standard manner, by the rules which are grounded in our biology.

Whether this biological purposing is innate... or whether it is derived via learning, mechanisms of concept formation, etc., it must *ultimately* derive its content from the details of our evolutionary history. So, unless doing arithmetic results from a total breakdown of the cognitive systems... then *whatever* you mean to do when you encounter "plus," that content has been determined by your experience coupled with evolutionary design (648-9).

Millikan derives a normative conclusion from her evolutionary biological solution.

She says that it provides a standard, how one ought to add or count, and which predicates one ought to use (or see as justified in using).

It is not clear to me that any evolutionary explanation can provide the kind of normative force that we ordinarily ascribe to mathematical claims, especially pure mathematical claims.

While we might defend an economy of free wood, or even of selling wood by the strength of the woodcutter, it seems that there are no real alternatives for measuring the wood.

We ought to multiply in particular ways, add in particular ways, and count in particular ways.

Biological purposiveness only supports rules which are conducive to survival.

Mathematical facts and rules may be, and are traditionally, seen as independent of evolutionary imperatives.

Of course, we might give up the traditional interpretation.

PS. When he was three, my son, Izzy, did exactly what Wittgenstein says at §279: he put his hand on his head and said, "I'm this tall!"